PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

A Glimpse of Nonlinear Phenomena from Chua's Oscillator

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A glimpse of nonlinear phenomena from Chua's oscillator

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Chua's oscillator is a simple electronic circuit whose (dimensionless) state equations are given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k\alpha(y - x - f(x)),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = k(x - y + z),$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = k(-\beta y - \gamma z),$$

where

$$f(x) = bx + \tfrac{1}{2}(a - b)[|x + 1| - |x - 1|].$$

It consists of two linear resistors, two linear capacitors, one linear inductor and one nonlinear resistor. Chua's circuit (which is Chua's oscillator with $\dot{\gamma} = 0$) can be built using discrete components (figure 1*a*) or as an integrated circuit (figure 1*b*). The speed at which the circuit operates can be set by choosing appropriate circuit component values. One of the advantages of Chua's oscillator is that the equations model the dynamical behaviour of the physical system quite accurately. By varying the six parameters (α, β, γ, a, b, k) of Chua's oscillator various nonlinear phenomena such as bifurcations, self-similarity, and chaos can be observed. Many attractors are found in Chua's oscillator by varying the parameters. Figure 2 shows a geometric model of Chua's double-scroll chaotic attractor which is observed in Chua's oscillator. By coupling several Chua's oscillators in an array even more complicated phenomena can be observed. Figure 6*a* shows spiral waves and target waves interacting in an array of Chua's oscillators. Figure 6*b* shows a Turing pattern which is observed in an array of Chua's oscillators.

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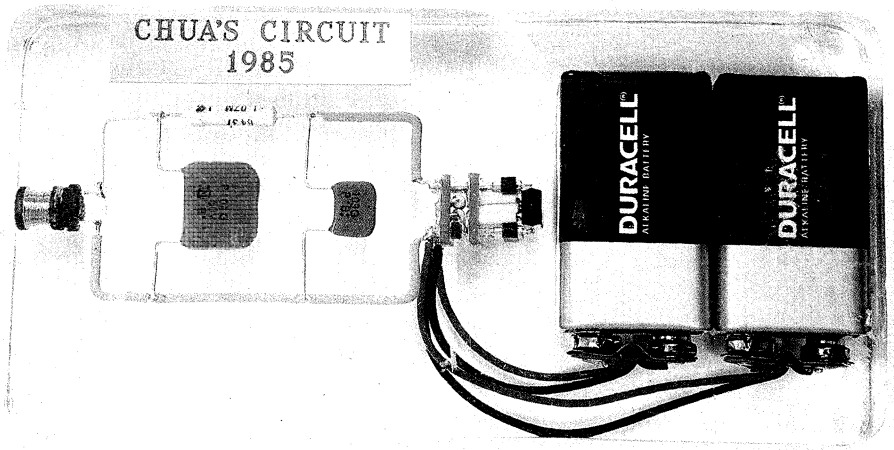

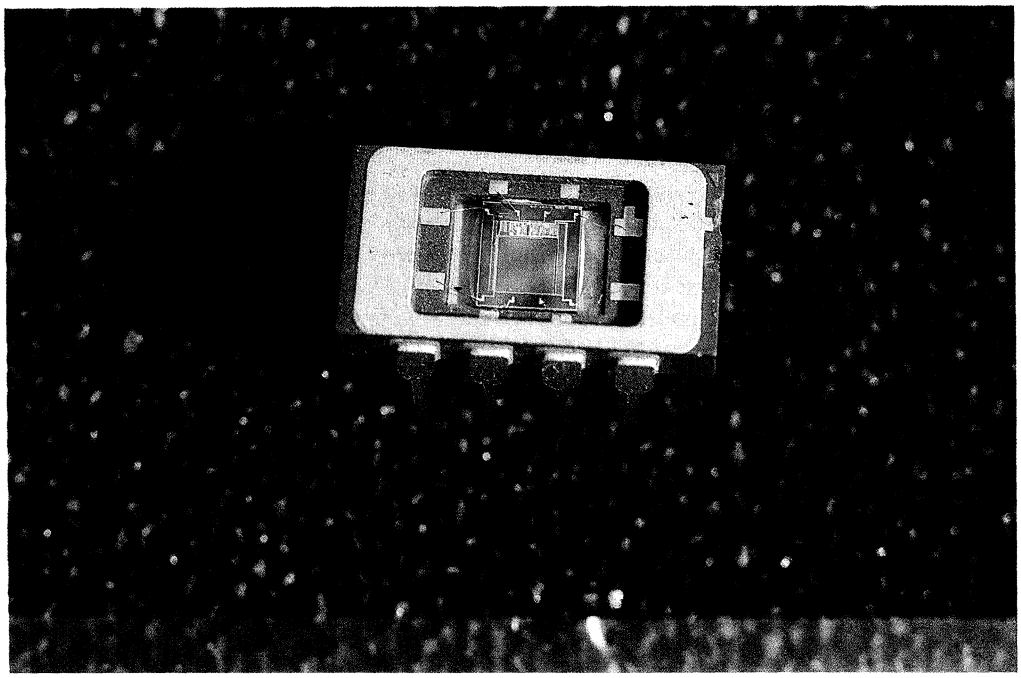

Figure 1. Chua's circuit can be built from discrete electronic components (top), or in a single chip via integrated circuit technology (bottom).

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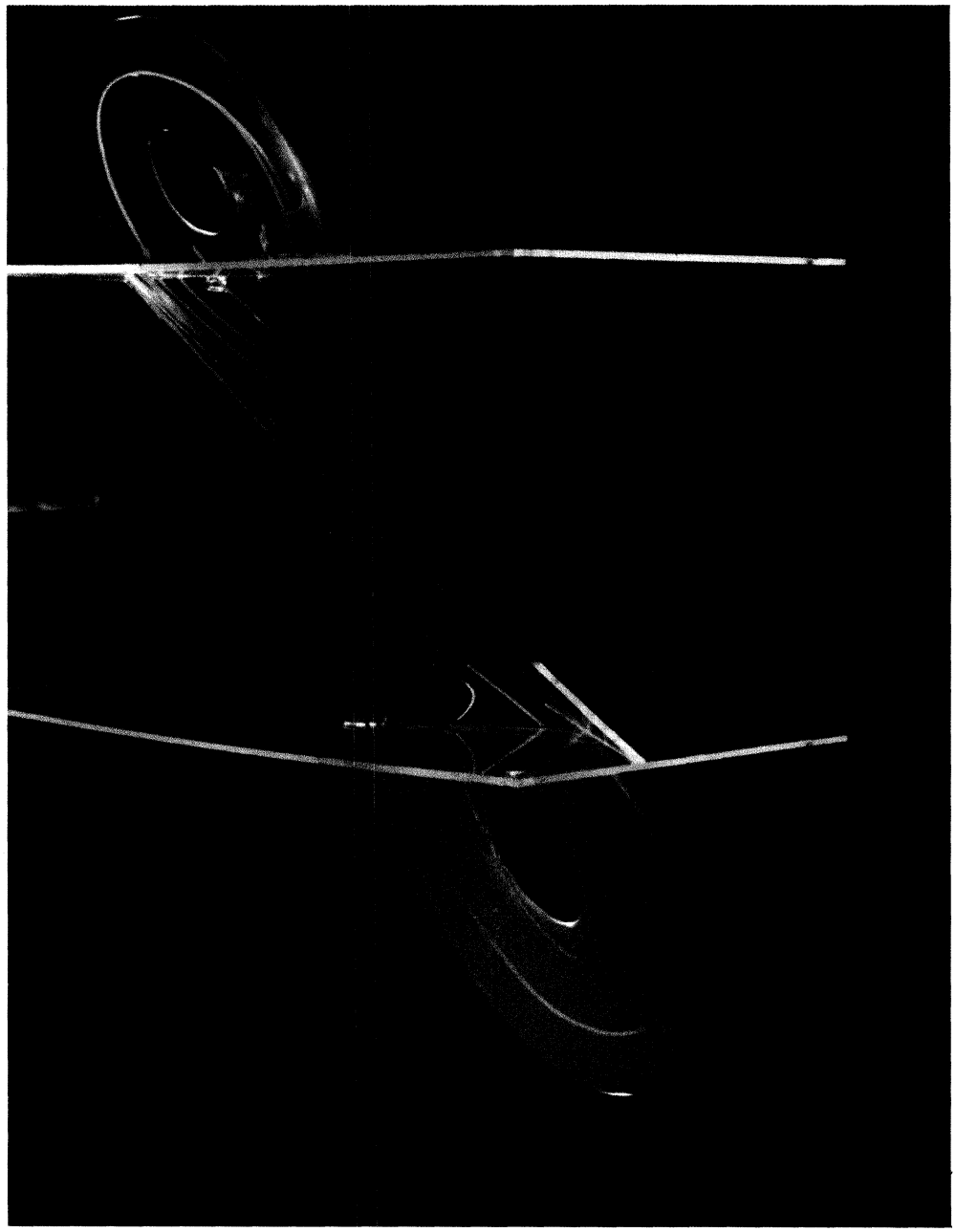

Figure 2. Three-dimensional fibre glass model of Chua's double-scroll attractor.

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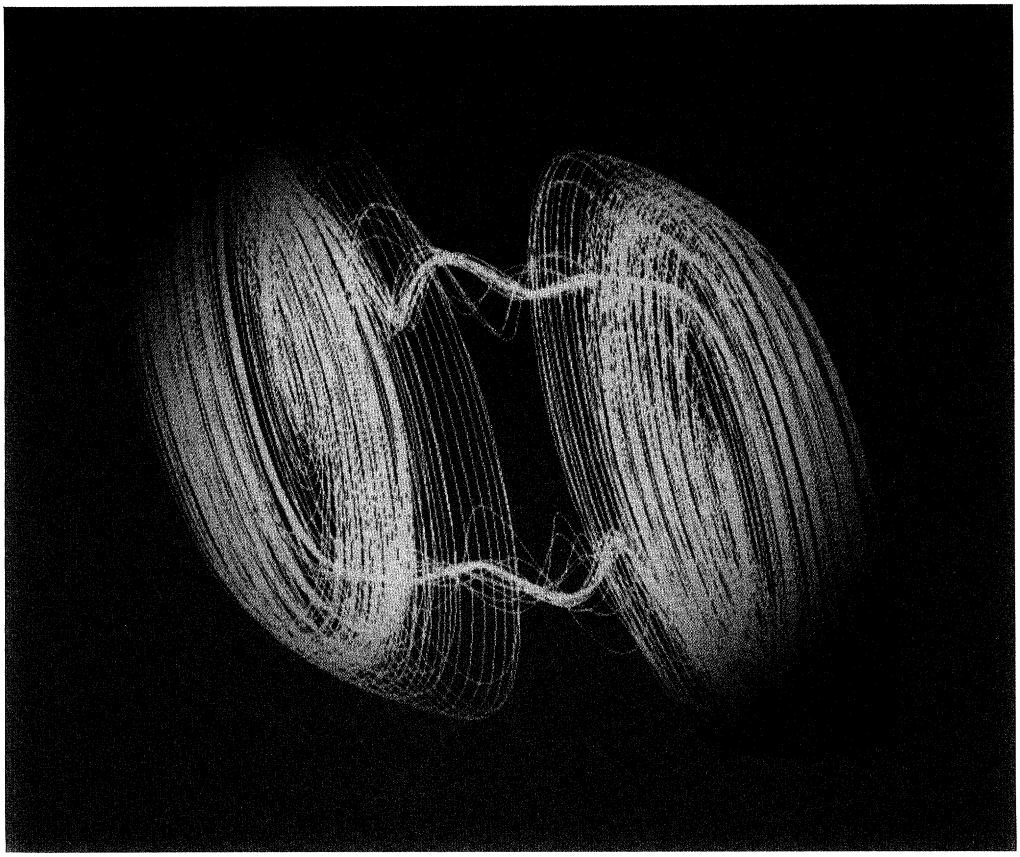

Figure 3. (*a*) A *strange attractor* from Chua's oscillator observed with the parameters:

α	β	γ	a	b	k
−1.301814	−0.0136073	−0.02969968	0.1690817	−0.4767822	1.0

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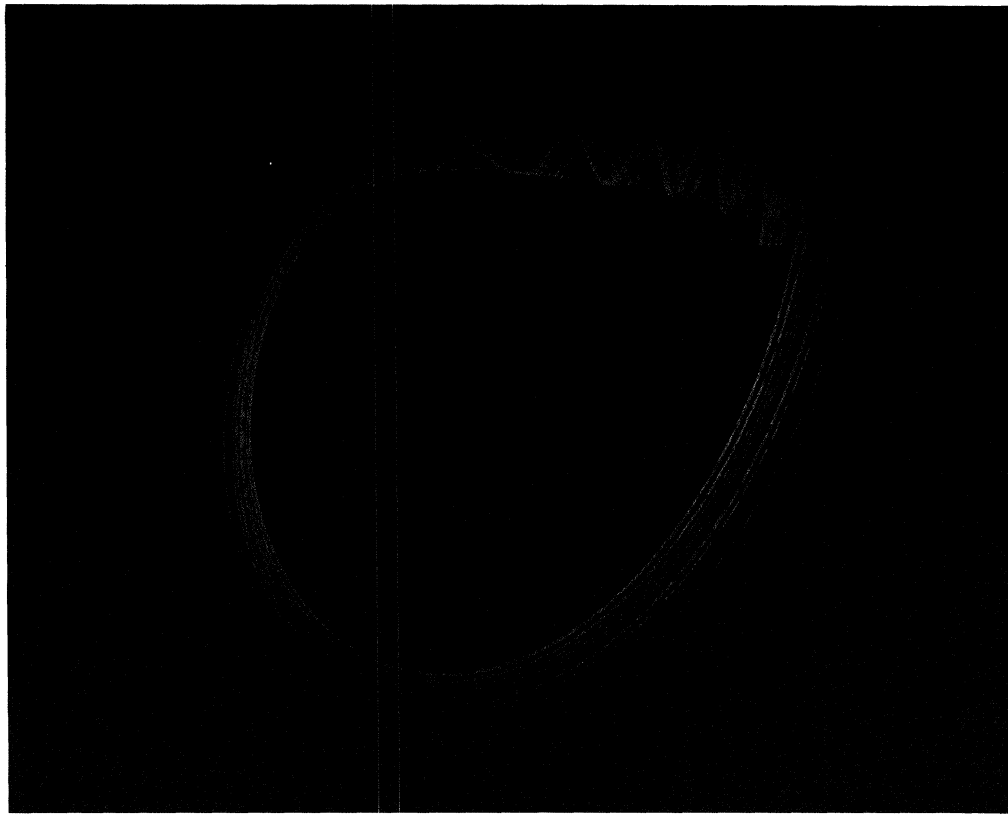

Figure 3. (*b*) A *strange attractor* from Chua's oscillator observed with the parameters:

α	β	γ	a	b	k
-1.0837792952	0.0000969088	0.0073276247	-0.0941189549	0.0001899298	-1.0

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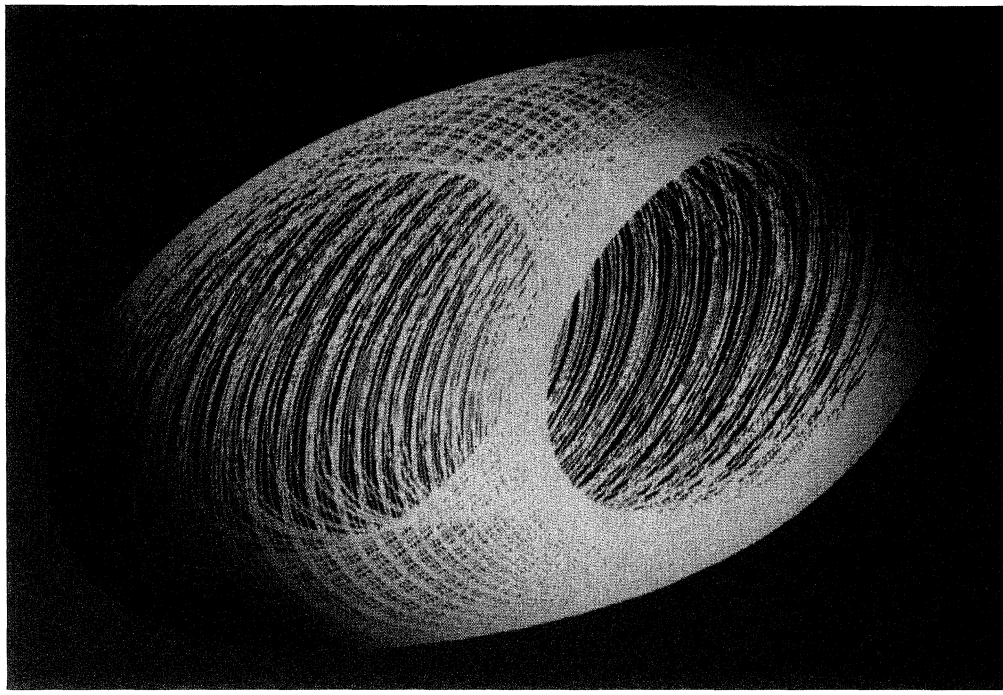

Figure 4. (*a*) A *strange attractor* from Chua's oscillator observed with the parameters:

α	β	γ	a	b	k
3.505	66.672752	−0.94779892	−0.855372	−1.09956	−1.0

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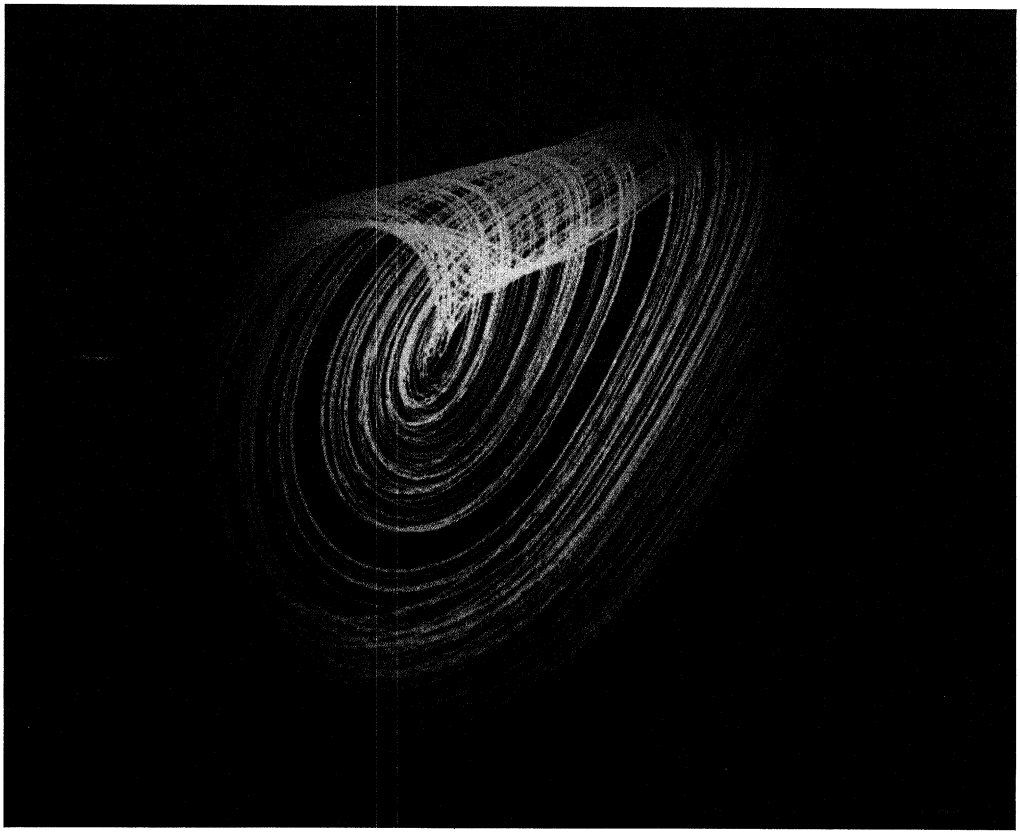

Figure 4. (*b*) A *strange attractor* from Chua's oscillator observed with the parameters:

α	β	γ	a	b	k
−6.69191	−1.52061	0.0	−1.142857	−0.7142857	1.0

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Figure 5. (*a*) A *strange attractor* from Chua's oscillator observed with the parameters:

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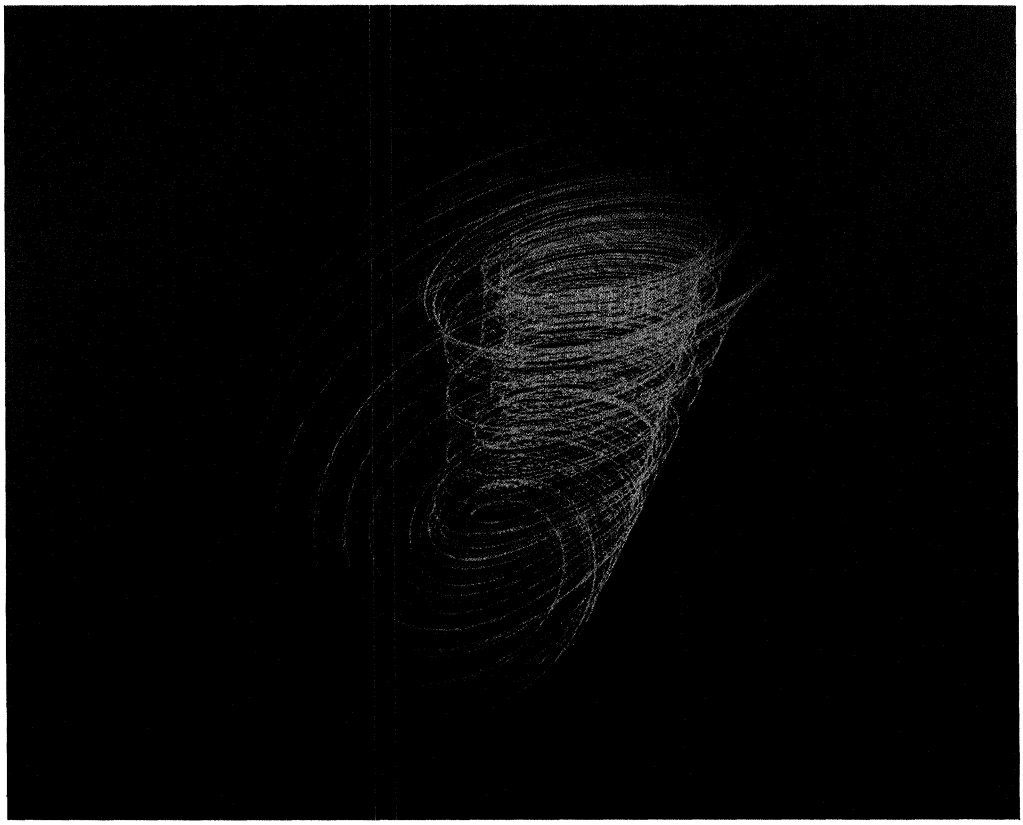

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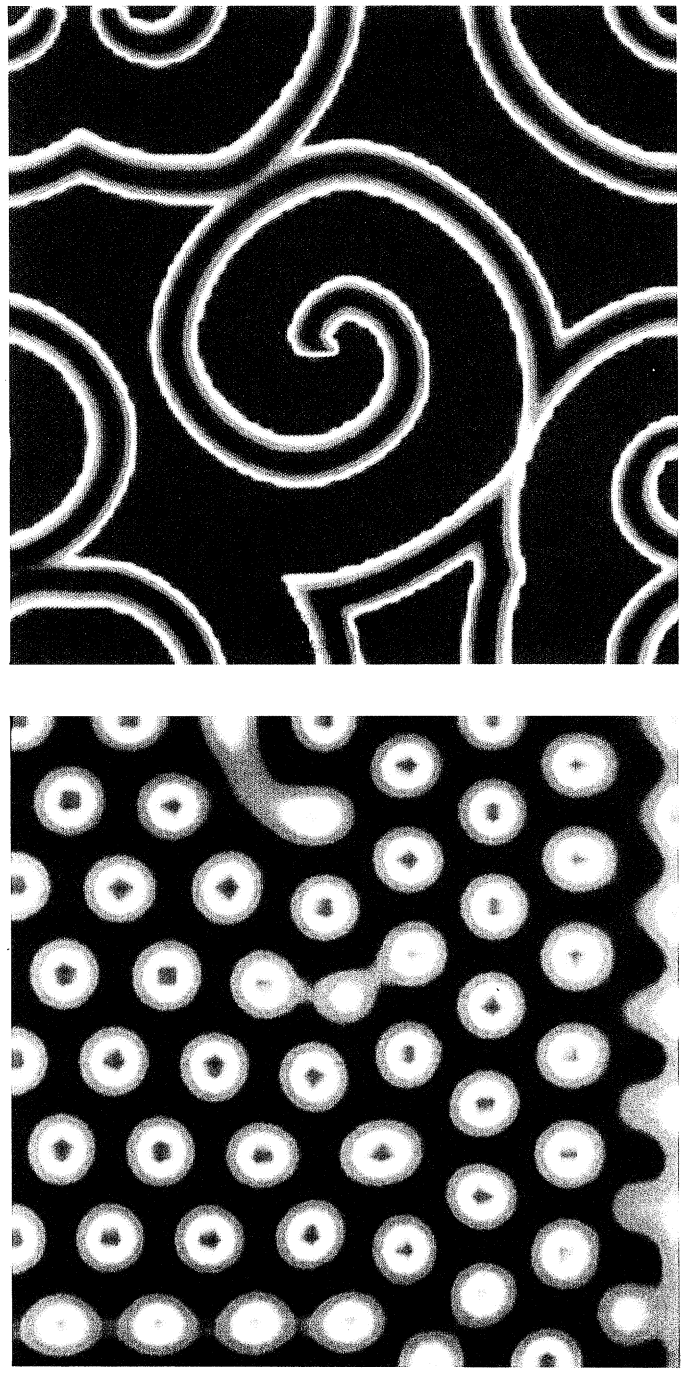

Figure 6. Snapshot depicting the interacting dynamics between *spiral* and *target* waves generated from a cellular neural network made of local interconnections of Chua's oscillators (top). A typical *Turing pattern* generated from an array of Chua's oscillators (bottom).

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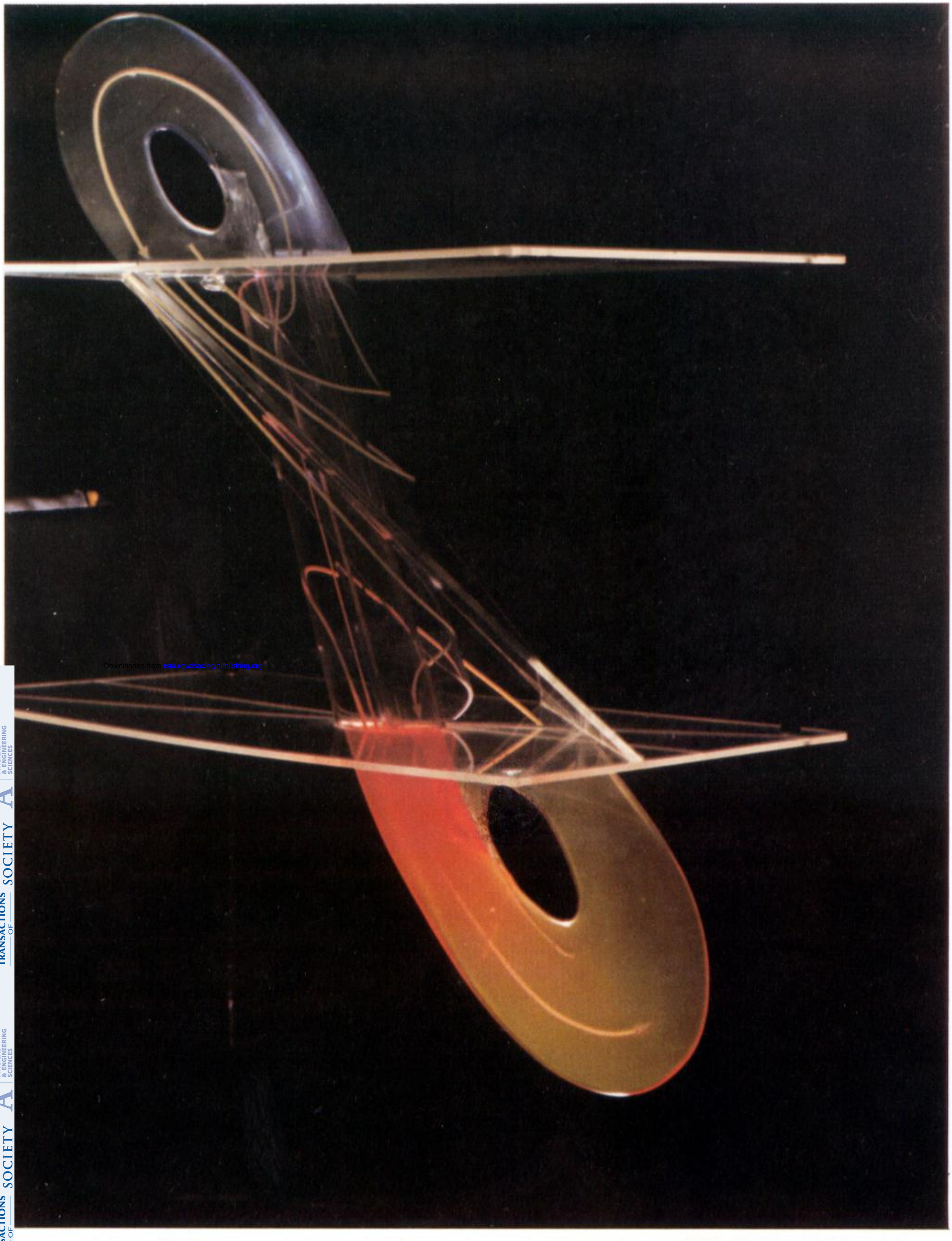

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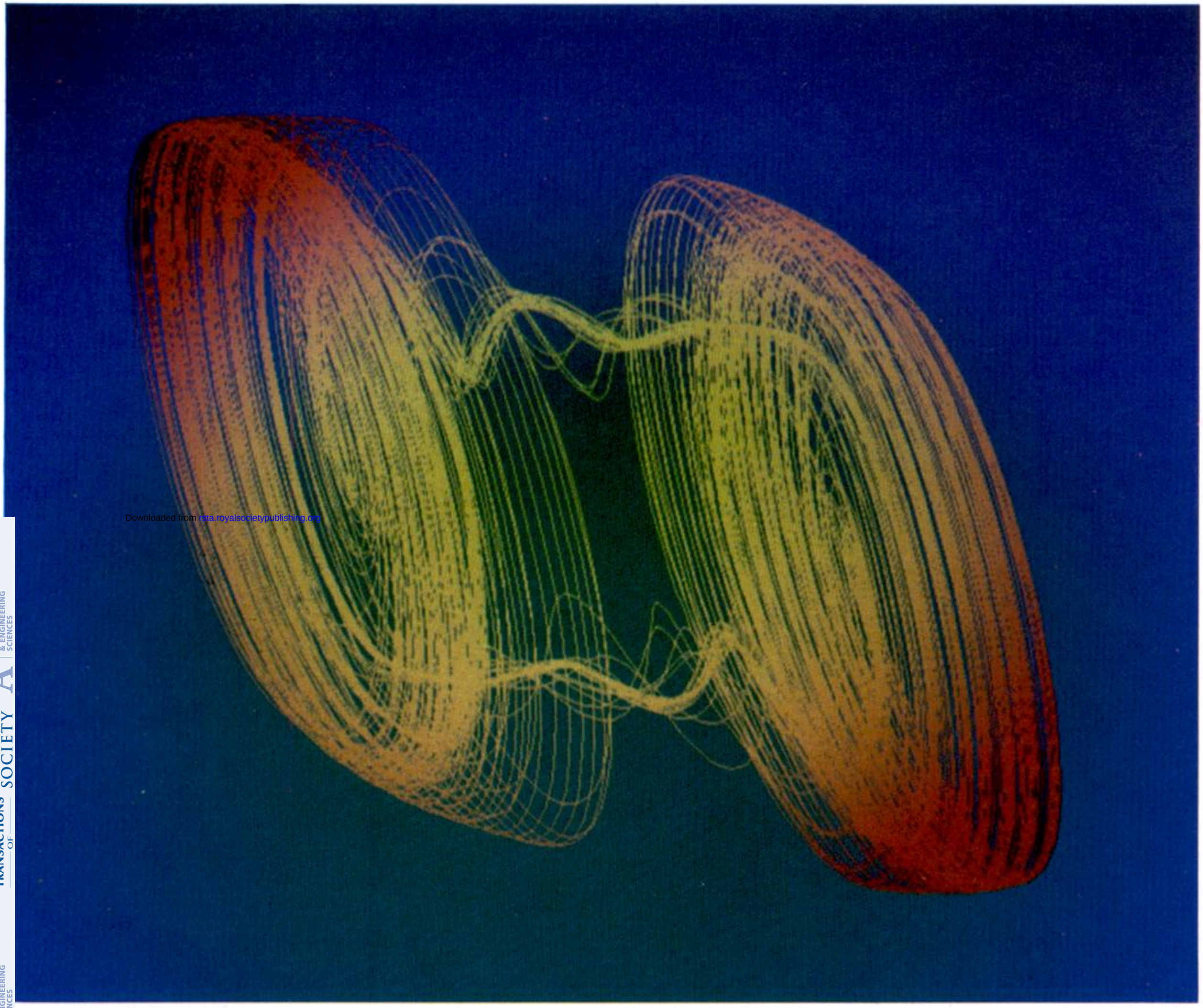

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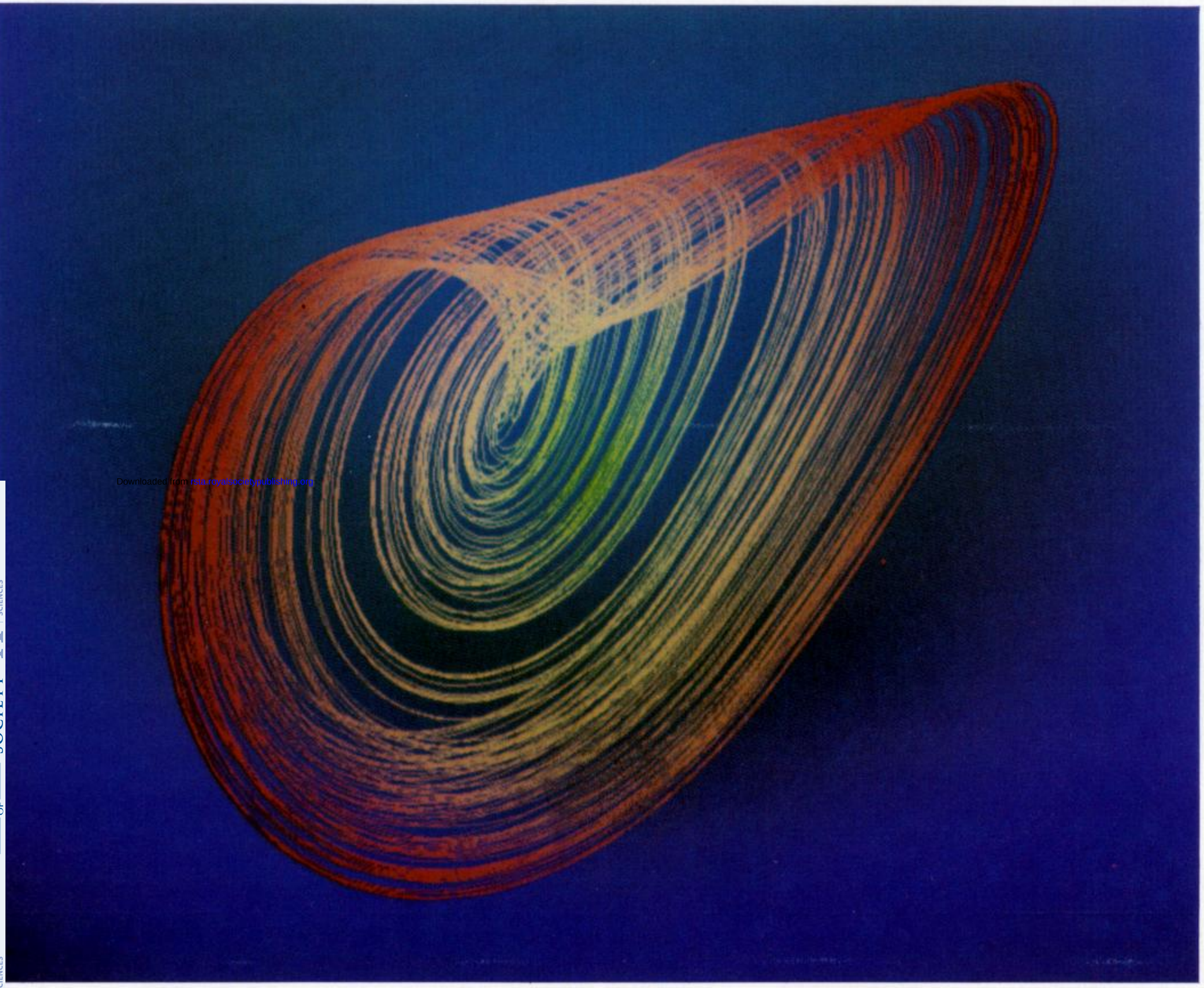

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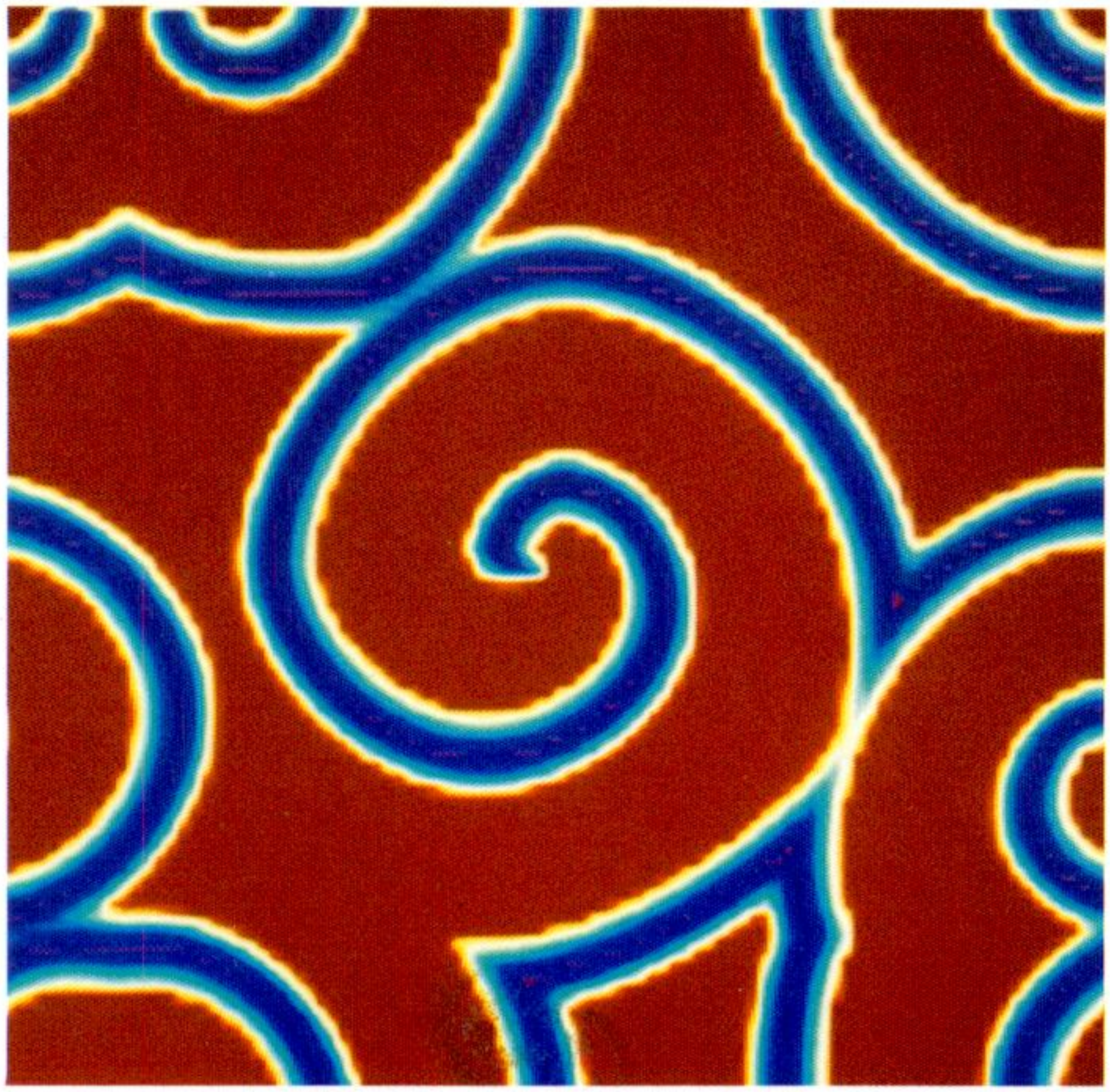

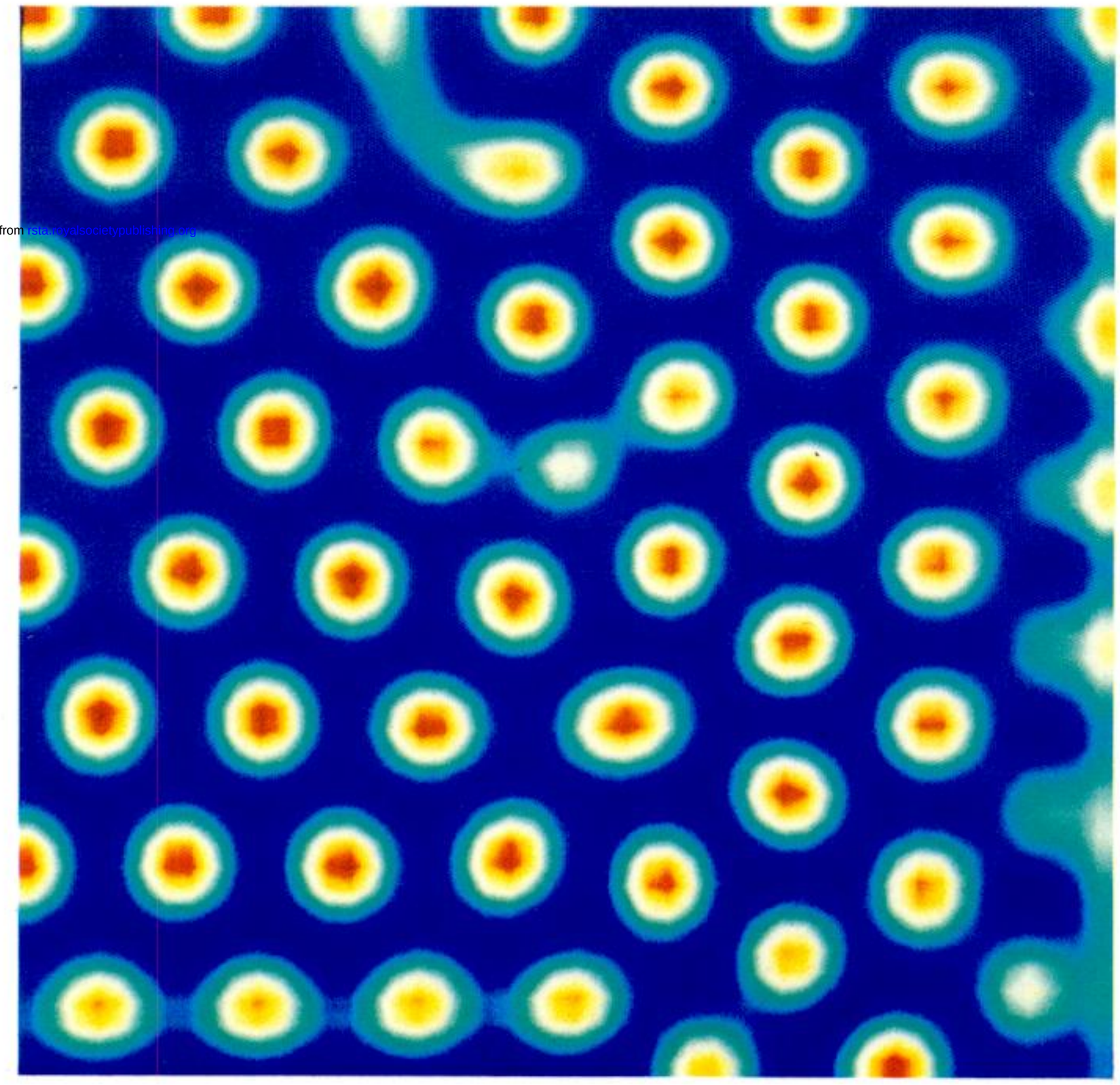

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